

09-05-07

DE



Attorney's Docket No. 042933/298965

PATENT

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

In re: Nativadade Lobo
Appl. No.: 09/625,201
Filed: July 21, 2000
For: PULSE SHAPING WHICH COMPENSATES FOR COMPONENT DISTORTION

Confirmation No.: 5615

BOX ISSUE FEE
Commissioner for Patents
P.O. Box 1450
Alexandria, VA 22313-1450

SUBMITTAL OF PRIORITY DOCUMENT

To complete the requirements of 35 U.S.C. § 119, enclosed is a certified copy of Great Britain priority Application No. 9801305.5, filed January 21, 1998.

Respectfully submitted,

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"Express Mail" mailing label number EV521113148US
Date of Deposit September 4, 2007

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Tamara Stevens

We show that it is possible to design pulses to be used by a modulator that incorporates the current superposition structure in such a way that

the BER, Spectral efficiency, amplitude envelope variation are traded off in a controlled way by ~~weighting~~

(i) developing separate cost functions for each (ii) weighting these ~~filters~~ costs

according to ~~input~~ design considerations

This is particularly desirable in the manufacture of mobile phones and designing ~~communication~~ systems

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Cardiff Road
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NP10 8QQ

I, the undersigned, being an officer duly authorised in accordance with Section 74(1) and (4) of the Deregulation & Contracting Out Act 1994, to sign and issue certificates on behalf of the Comptroller-General, hereby certify that annexed hereto is a true copy of the documents as originally filed in connection with patent application GB9801305.5 filed on 21 January 1998.

In accordance with the Patents (Companies Re-registration) Rules 1982, if a company named in this certificate and any accompanying documents has re-registered under the Companies Act 1980 with the same name as that with which it was registered immediately before re-registration save for the substitution as, or inclusion as, the last part of the name of the words "public limited company" or their equivalents in Welsh, references to the name of the company in this certificate and any accompanying documents shall be treated as references to the name with which it is so re-registered.

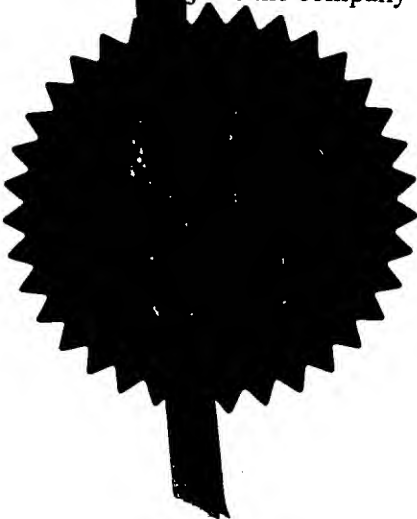
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Re-registration under the Companies Act does not constitute a new legal entity but merely subjects the company to certain additional company law rules.

Signed



Dated 21 August 2007



Request for grant of a patent

(See the notes on the back of this form. You can also get an explanatory leaflet from the Patent Office to help you fill in this form)



The Patent Office

Cardiff Road
Newport
Gwent NP9 1RH

1. Your reference

PAT 98003 GB

2. Patent application number
(The Patent Office will fill in the)

9801305.5

21 JAN 1998

3. Full name, address and postcode of the or of each applicant (underline all surnames)

NOKIA MOBILE PHONES LIMITED
KEILALAHDENTIE 4
02150 ESPOO
FINLAND

Patents ADP number (if you know it)

If the applicant is a corporate body, give the country/state of its incorporation

FINLAND

5911995004

4. Title of the invention

OPTIMIZER

5. Name of your agent (if you have one)

"Address for service" in the United Kingdom to which all correspondence should be sent (including the postcode)

MRS HELEN LOUISE HAWS
NOKIA MOBILE PHONES
PATENT DEPARTMENT
ST GEORGES COURT
ST GEORGES ROAD
CAMBERLEY
SURREY GU15 3QZ UK

Patents ADP number (if you know it)

4105177001

6. If you are declaring priority from one or more earlier patent applications, give the country and the date of filing of the or of each of these earlier applications and (if you know it) the or each application number

Country

Priority application number
(if you know it)

Date of filing
(day / month / year)

7. If this application is divided or otherwise derived from an earlier UK application, give the number and the filing date of the earlier application

Number of earlier application

Date of filing
(day / month / year)

8. Is a statement of inventorship and of right to grant of a patent required in support of this request? (Answer 'Yes' if:

YES

- a) any applicant named in part 3 is not an inventor, or
 - b) there is an inventor who is not named as an applicant, or
 - c) any named applicant is a corporate body.
- See note (d))

I certify this to be a true copy.

G D Court
Acting for Comptroller

Patents Form 1/77

9. Enter the number of sheets for any of the following items you are filing with this form. Do not count copies of the same document

Continuation sheets of this form

Description 17

Claim(s)

Abstract

Drawing(s) IN WITH TEXT

10. If you are also filing any of the following, state how many against each item.

Priority documents

Translations of priority documents

Statement of inventorship and right to grant of a patent (Patents Form 7/77)

Request for preliminary examination and search (Patents Form 9/77)

Request for substantive examination (Patents Form 10/77)

Any other documents (please specify)

11.

I/We request the grant of a patent on the basis of this application.

Signature

H L HAWS

AGENT FOR THE APPLICANT

Date 21 JANUARY 1998

12. Name and daytime telephone number of person to contact in the United Kingdom

HELEN HAWS 01276 419346

Warning

After an application for a patent has been filed, the Comptroller of the Patent Office will consider whether publication or communication of the invention should be prohibited or restricted under Section 22 of the Patents Act 1977. You will be informed if it is necessary to prohibit or restrict your invention in this way. Furthermore, if you live in the United Kingdom, Section 23 of the Patents Act 1977 stops you from applying for a patent abroad without first getting written permission from the Patent Office unless an application has been filed at least 6 weeks beforehand in the United Kingdom for a patent for the same invention and either no direction prohibiting publication or communication has been given, or any such direction has been revoked.

Notes

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- If you have answered 'Yes' Patents Form 7/77 will need to be filed.
- Once you have filled in the form you must remember to sign and date it.
- For details of the fee and ways to pay please contact the Patent Office.


```
Receiver[L][tom2, StartingQuadrant -> 4]
```

```
{1, 1, 1, 1, 1, 1, -1, -1, -1, 1, 1, -1, 1, -1, 1, -1, -1, -1, -1, 1, 1, 1, -1, -1,
-1, 1, 1, 1, -1, 1, -1, -1, 1, -1, -1, 1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, 1,
1, 1, 1, -1, 1, -1, 1, -1, 1, -1, 1, 1, -1, -1, 1, -1, 1, 1, 1, 1, 1, -1,
-1, -1, 1, -1, 1, 1, 1, -1, 1, -1, -1, 1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, -1, -1}
```

We write all the possible terms

```
J^a0, n-5 ( J^(aN-4+aN-3+aN-2+aN-1+aN) Pulse[0][δT] +
J^(aN-4+aN-3+aN-2+aN-1) Pulse[0][T+δT] + J^(aN-4+aN-3+aN-2) Pulse[0][2T+δT] +
J^(aN-4+aN-3) Pulse[0][3T+δT] + J^aN-4 Pulse[0][4T+δT] + Pulse[0][5T+δT] +
J^(-aN-5) Pulse[0][6T+δT] + J^(-aN-5-aN-6) Pulse[0][7T+δT] +
J^(-aN-5-aN-6-aN-7) Pulse[0][8T+δT] + J^(aN-4+aN-3+aN-2+aN-1+aN-aN-1) Pulse[1][δT] +
J^(aN-4+aN-3+aN-2+aN-1-aN-2) Pulse[1][T+δT] + J^(aN-4+aN-3+aN-2+aN-3) Pulse[1][2T+δT] +
J^(aN-4+aN-3-aN-4) Pulse[1][3T+δT] + J^(aN-4-aN-5) Pulse[1][4T+δT] +
J^(-aN-6) Pulse[1][5T+δT] + J^(-aN-5-aN-7) Pulse[1][6T+δT] )

( J^(aN-4+aN-3+aN-2+aN-1+aN) Pulse[0][δT] +
J^(aN-4+aN-3+aN-2+aN-1) Pulse[0][T+δT] + J^(aN-4+aN-3+aN-2) Pulse[0][2T+δT] +
J^(aN-4+aN-3) Pulse[0][3T+δT] + J^aN-4 Pulse[0][4T+δT] + Pulse[0][5T+δT] +
J^(-aN-5) Pulse[0][6T+δT] + J^(-aN-5-aN-6) Pulse[0][7T+δT] +
J^(-aN-5-aN-6-aN-7) Pulse[0][8T+δT] + J^(aN-4+aN-3+aN-2+aN-1+aN-aN-1) Pulse[1][δT] +
J^(aN-4+aN-3+aN-2+aN-1-aN-2) Pulse[1][T+δT] +
J^(aN-4+aN-3+aN-2+aN-3) Pulse[1][2T+δT] +
J^(aN-4+aN-3-aN-4) Pulse[1][3T+δT] + J^(aN-4-aN-5) Pulse[1][4T+δT] +
J^(-aN-6) Pulse[1][5T+δT] + J^(-aN-5-aN-7) Pulse[1][6T+δT] ) /. {aN -> x0,
aN-1 -> x1, aN-2 -> x2, aN-3 -> x3, aN-4 -> x4, aN-5 -> x5, aN-6 -> x6, aN-7 -> x7}

J^x0+x1+x2+x3+x4 Pulse[0][δT] + J^x1+x2+x3+x4 Pulse[0][T+δT] +
J^x2+x3+x4 Pulse[0][2T+δT] + J^x3+x4 Pulse[0][3T+δT] + J^x4 Pulse[0][4T+δT] +
Pulse[0][5T+δT] + J^-x5 Pulse[0][6T+δT] + J^-x5-x6 Pulse[0][7T+δT] +
J^-x5-x6-x7 Pulse[0][8T+δT] + J^x0+x2+x3+x4 Pulse[1][δT] + J^x1+x3+x4 Pulse[1][T+δT] +
J^x2+x3+x4 Pulse[1][2T+δT] + J^x3 Pulse[1][3T+δT] + J^x4-x5 Pulse[1][4T+δT] +
J^-x6 Pulse[1][5T+δT] + J^-x5+x7 Pulse[1][6T+δT]

Clear[AbsoluteValue]

AbsoluteValue[Pulse_][{x0_, x1_, x2_, x3_, x4_, x5_, x6_, x7_}][δT_] :=
Module[{a1, a2, a3, a4},
a1 = J^x0+x1+x2+x3+x4 Pulse[0][δT] +
J^x1+x2+x3+x4 Pulse[0][T+δT] + J^x2+x3+x4 Pulse[0][2T+δT] + J^x3+x4 Pulse[0][3T+δT] +
J^x4 Pulse[0][4T+δT] + Pulse[0][5T+δT] + J^-x5 Pulse[0][6T+δT] +
J^-x5-x6 Pulse[0][7T+δT] + J^-x5-x6-x7 Pulse[0][8T+δT] + J^x0+x2+x3+x4 Pulse[1][δT] +
J^x1+x3+x4 Pulse[1][T+δT] + J^x2+x3+x4 Pulse[1][2T+δT] + J^x3 Pulse[1][3T+δT] +
J^x4-x5 Pulse[1][4T+δT] + J^-x6 Pulse[1][5T+δT] + J^-x5+x7 Pulse[1][6T+δT];
a2 = ( Re[a1]^2 + Im[a1]^2 // ComplexExpand) /. {Im[x_] -> 0, Re[x_] -> x};
a2 /. x_?NumberQ y_ -> Round[x] y]

AbsoluteValueInterfearingregions[Pulse_][
{x0_, x1_, x2_, x3_, x4_, x5_, x6_, x7_}][δT_] :=
Module[{a1, a2, a3, a4},
a1 = J^x0+x1+x2+x3+x4 Pulse[0][δT] + J^x2+x3+x4 Pulse[0][2T+δT] + J^-x5 Pulse[0][6T+δT] +
J^-x5-x6-x7 Pulse[0][8T+δT] + J^x1+x3+x4 Pulse[1][T+δT] + J^-x6 Pulse[1][5T+δT];
a2 = ( Re[a1]^2 + Im[a1]^2 // ComplexExpand) /. {Im[x_] -> 0, Re[x_] -> x};
a2 /. x_?NumberQ y_ -> Round[x] y]
```

AbsoluteValue[Pulse[8]][{-1, -1, -1, -1, -1, -1, -1, -1}][0.5 T]

$$\begin{aligned} & x0[4]^2 + x0[12]^2 - 2 x0[4] x0[20] + x0[20]^2 - 2 x0[12] x0[28] + x0[28]^2 + 2 x0[4] x0[36] - \\ & 2 x0[20] x0[36] + x0[36]^2 + 2 x0[12] x0[44] - 2 x0[28] x0[44] + x0[44]^2 - 2 x0[4] x0[52] + \\ & 2 x0[20] x0[52] - 2 x0[36] x0[52] + x0[52]^2 - 2 x0[12] x0[60] + 2 x0[28] x0[60] - \\ & 2 x0[44] x0[60] + x0[60]^2 + 2 x0[4] x0[68] - 2 x0[20] x0[68] + 2 x0[36] x0[68] - \\ & 2 x0[52] x0[68] + x0[68]^2 + 2 x0[12] x1[4] - 2 x0[28] x1[4] + 2 x0[44] x1[4] - \\ & 2 x0[60] x1[4] + x1[4]^2 - 2 x0[4] x1[12] + 2 x0[20] x1[12] - 2 x0[36] x1[12] + \\ & 2 x0[52] x1[12] - 2 x0[68] x1[12] + x1[12]^2 + 2 x0[12] x1[20] - 2 x0[28] x1[20] + \\ & 2 x0[44] x1[20] - 2 x0[60] x1[20] + 2 x1[4] x1[20] + x1[20]^2 + 2 x0[4] x1[28] - \\ & 2 x0[20] x1[28] + 2 x0[36] x1[28] - 2 x0[52] x1[28] + 2 x0[68] x1[28] - 2 x1[12] x1[28] + \\ & x1[28]^2 + 2 x0[12] x1[36] - 2 x0[28] x1[36] + 2 x0[44] x1[36] - 2 x0[60] x1[36] + \\ & 2 x1[4] x1[36] + 2 x1[20] x1[36] + x1[36]^2 - 2 x0[4] x1[44] + 2 x0[20] x1[44] - \\ & 2 x0[36] x1[44] + 2 x0[52] x1[44] - 2 x0[68] x1[44] + 2 x1[12] x1[44] - \\ & 2 x1[28] x1[44] + x1[44]^2 + 2 x0[12] x1[52] - 2 x0[28] x1[52] + 2 x0[44] x1[52] - \\ & 2 x0[60] x1[52] + 2 x1[4] x1[52] + 2 x1[20] x1[52] + 2 x1[36] x1[52] + x1[52]^2 \end{aligned}$$

We set $T = 1$ to avoid precision problems.

$T := 1$

PulseSampling := $T / 8$

var = Join[Table[x0[i], {i, 0, LaurentLK[L][0] T / PulseSampling - 1}],
Table[x1[i], {i, 0, LaurentLK[L][1] T / PulseSampling - 1}]];


```

initsol = Join[Table[LaurentC[8][0][i PulseSampling],
  {i, 0, LaurentLK[L][0] T / PulseSampling - 1}], Table[
  LaurentC[8][1][i PulseSampling], {i, 0, LaurentLK[L][1] T / PulseSampling - 1}]];
$Aborted

```

```

hessian = IdentityMatrix[Length[var]];

Pulse[8][0][y_] := x0[Round[y / PulseSampling]];
Pulse[8][1][y_] := x1[Round[y / PulseSampling]];

TempFun[x_][y_] := (AbsoluteValue[Pulse[8]][x][y] - 1)^2 // Expand;

TempFun2[x_][y_] := (AbsoluteValueInterfearingregions[Pulse[8]][x][y]);

TempFun[{-1, -1, -1, -1, -1, -1, -1, -1}][0.5 T];

ConvertToBitSeq[Depth_][Num_] :=
Module[{bit, n},
  n = Num;
  ConvertNext := (bit = Mod[n, 2]; n = Floor[n / 2]; bit);
  (Table[ConvertNext, {i, 1, Depth}] // Reverse) /. {0 -> -1});

PossSeq = Table[i, {i, 1, 2^8}] // Map[ConvertToBitSeq[8], #]&;

```

We calculate the

```

CostGenerator[TempFun_] :=
Module[{Cost},
  Cost = 0;
  PulseValue = 0;
  CostUpdate := (Cost =
    Cost + Map[TempFun[#][PulseValue]&, PossSeq] // Apply[Plus, #]& // # / 256 &;
    PulseValue = PulseValue + PulseSampling);
  Table[CostUpdate, {i, 1, T / PulseSampling // Round}]; Cost];

AmplitudeCost = CostGenerator[TempFun] * PulseSampling;

BERCost = CostGenerator[TempFun2] * PulseSampling;

Save["AmplitudeCost.m", {BERCost, AmplitudeCost, PulseSampling, T}]

```

In this section we find the bandwidth of the signal
Derivative of the Sync function is represented by

```
Clear[T, PulseSampling]
```

```

D[ $\frac{\sin[x]}{x}$ , x]
 $\frac{\cos[x]}{x} - \frac{\sin[x]}{x^2}$ 
DSinc[0.0] := 0
DSinc[x_] /; x != 0 :=  $\frac{\cos[x]}{x} - \frac{\sin[x]}{x^2}$ 
<< AmplitudeCost.m;

```

The bandwidth is given by

```

TempInt0[m_, n_] :=
  ( $\frac{\pi}{\text{PulseSampling}}$ )2 NIntegrate[DSinc[ $\frac{\pi}{\text{PulseSampling}}$  (x - m PulseSampling)]
    DSinc[ $\frac{\pi}{\text{PulseSampling}}$  (x - n PulseSampling)], {x, 0, 9 T}]
TempInt1[m_, n_] :=
  ( $\frac{\pi}{\text{PulseSampling}}$ )2 NIntegrate[DSinc[ $\frac{\pi}{\text{PulseSampling}}$  (x - m PulseSampling)]
    DSinc[ $\frac{\pi}{\text{PulseSampling}}$  (x - n PulseSampling)],
    {x, 0, 7 T}]
Table[TempInt0[0, n], {n, 0, 5}]
General::stop :
  Further output of NIntegrate::ncvb will be suppressed during this calculation.
{1.27879×107, 5.06325, 10.6271, 9.78983, 4.51144, -2.28746}
PulseSampling
 $\frac{1}{8}$ 
ApproxBandWidth = Sum[(x0[i + 1] - x0[i])2 / PulseSampling, {i, 0, 70}] +
  Sum[(x1[i + 1] - x1[i])2 / PulseSampling, {i, 0, 54}];
BandWidth = Sum[x0[i] x0[j] TempInt0[i, j], {i, 0, 71}, {j, 0, 71}] +
  Sum[x1[i] x1[j] TempInt1[i, j], {i, 0, 55}, {j, 0, 55}];
Save["BandWidthCost.m", {BandWidth, T,}]
BandWidthCost = (ApproxBandWidth - 0.6)2;

```

```

BERWeight = 0.3
BandWidthWeight = 0.4;
AmplitudeWeight = 1.0 - BandWidthWeight - BERWeight;

0.3

TotalCost = BandWidthWeight BandWidthCost + AmplitudeWeight AmplitudeCost +
  BERWeight * BERCOST;

dir = Directory[];
SetDirectory["../new2satellite"];
<< DavidonFletcherPowell.m
SetDirectory[dir];

L := 8

DavidonFletcherPowell[TotalCost, var, hessian, initsol, 10];

{0.0207507, {DavidonFletcherPowell`Private`alpha$39228→2.77153}}
{0.0164912, {DavidonFletcherPowell`Private`alpha$39228→2.45972}}
{0.0142842, {DavidonFletcherPowell`Private`alpha$39228→1.88521}}
{0.0126876, {DavidonFletcherPowell`Private`alpha$39228→2.08508}}
{0.0117343, {DavidonFletcherPowell`Private`alpha$39228→1.92086}}
{0.0112982, {DavidonFletcherPowell`Private`alpha$39228→1.55191}}
{0.0110916, {DavidonFletcherPowell`Private`alpha$39228→1.44356}}
{0.010974, {DavidonFletcherPowell`Private`alpha$39228→1.63222}}
{0.010886, {DavidonFletcherPowell`Private`alpha$39228→2.1399}}
{0.0108318, {DavidonFletcherPowell`Private`alpha$39228→2.14114}}

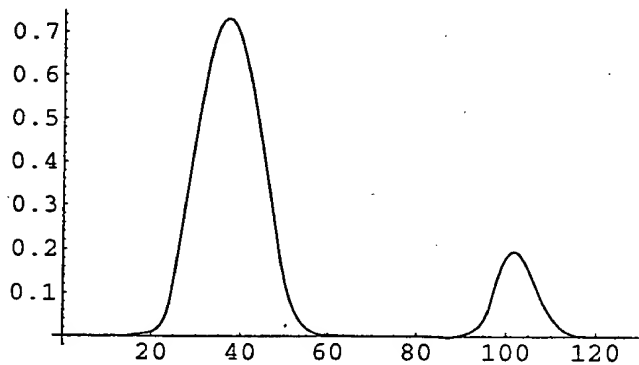
temp0[t_] = LaurentC[8][0][t];

temp1[t_] = LaurentC[8][1][t];

initsolC =
Join[Table[temp0[i PulseSampling], {i, 0, LaurentLK[L][0] T/PulseSampling - 1}],
  Table[temp1[i PulseSampling], {i, 0, LaurentLK[L][1] T/PulseSampling - 1}]];

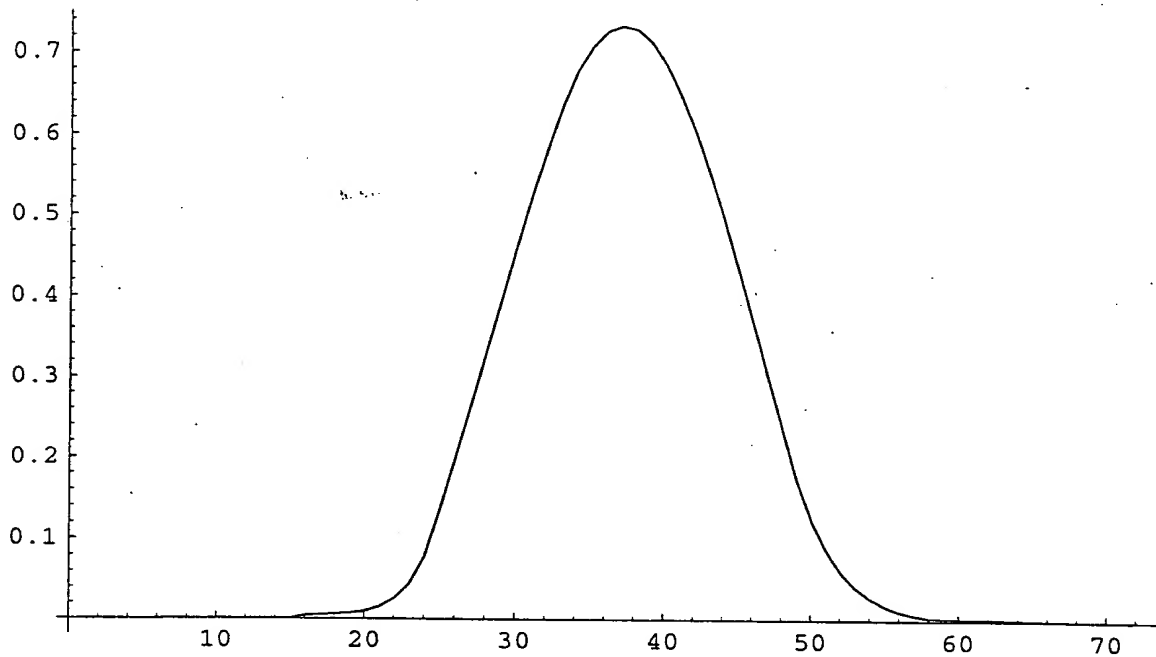
ListPlot[%278[[2]], PlotJoined -> True, PlotRange -> All]

```



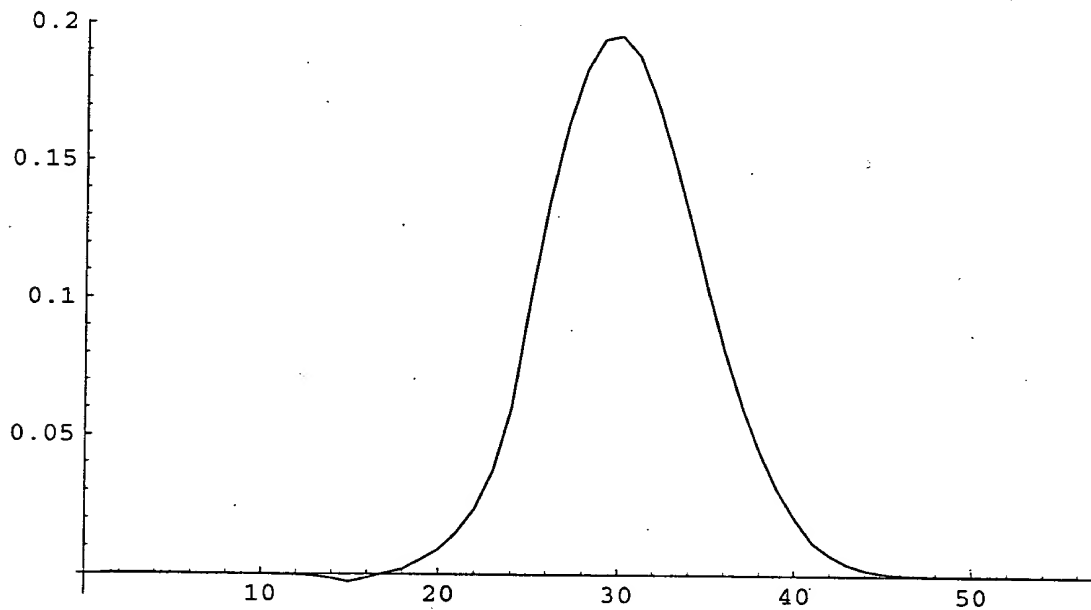
- Graphics -

```
ListPlot[%278[[2]] // Take[#, 72]&, PlotJoined -> True, PlotRange -> All]
```



- Graphics -

```
ListPlot[%278[[2]] // Take[#, -56]&, PlotJoined -> True, PlotRange -> All]
```



- Graphics -

```
{AmplitudeCost, BERCost, BandWidthCost, ApproxBandWidth} /. Thread[var -> initsol]
```

```
{0.0687008, 0.00375044, 0.0126982, 0.712686}
```

```
{AmplitudeCost, BERCost, BandWidthCost, ApproxBandWidth} /. Thread[var -> %278[[2]]]
```

```
{0.0208114, 0.00872422, 0.00492789, 0.670199}
```

```
(ApproxBandWidth - 0.6)^2 /. Thread[var -> initsol]
```

```
0.0126982
```

```
(ApproxBandWidth - 0.6)^2 /. Thread[var -> %278[[2]]]
```

```
0.00492789
```

```
OptPulse[8][0] = Interpolation[
  Table[{PulseSampling i, x0[i]}, {i, 0, LaurentLK[L][0] T / PulseSampling - 1}] /.
  Thread[var -> %278[[2]]]
```

```
InterpolatingFunction[{{0, 8.875}}, <>]
```

```
OptPulse[8][1] = Interpolation[
  Table[{PulseSampling i, x1[i]}, {i, 0, LaurentLK[L][1] T / PulseSampling - 1}] /.
  Thread[var -> %278[[2]]]
```

```
InterpolatingFunction[{{0, 6.875}}, <>]
```

```
tom3 = Modulator[L][RandomBitSeq, NumberOfCurves -> 2, ModulatingPulse -> OptPulse];
```

```
InterpolatingFunction::dmval : Input value  $\left\{\frac{221}{32}\right\}$  lies outside the range of data
in the interpolating function. Extrapolation will be used.
```

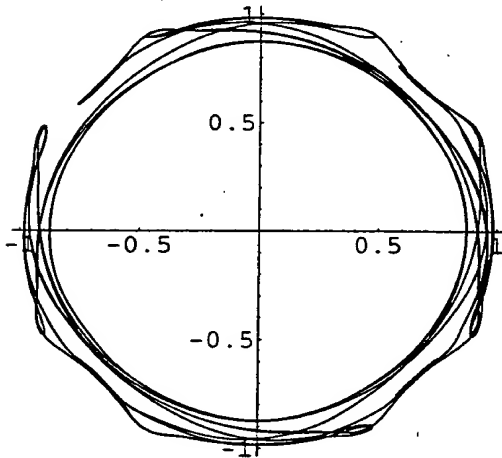
```
InterpolatingFunction::dmval : Input value  $\left\{\frac{285}{32}\right\}$  lies outside the range of data
in the interpolating function. Extrapolation will be used.
```

```
InterpolatingFunction::dmval : Input value  $\left\{\frac{111}{16}\right\}$  lies outside the range of data
in the interpolating function. Extrapolation will be used.
```

```
General::stop :
```

```
Further output of InterpolatingFunction::dmval will be suppressed during this calculation.
```

```
ListPlot[{Re[tom3], Im[tom3]} // Transpose, PlotJoined -> True, AspectRatio -> 1]
```



```
- Graphics -
```

```
{AmplitudeCost, BERCost, BandWidthCost, ApproxBandWidth} /. Thread[var -> initsol]
```

```
{0.0687008, 0.00375044, 0.0126982, 0.712686}
```

```
{AmplitudeCost, BERCost, BandWidthCost, ApproxBandWidth} /. Thread[var -> %278[[2]]]
```

```
{0.0208114, 0.00872422, 0.00492789, 0.670199}
```

```
Plot[%298[t], {t, 0, 7 T}, PlotRange -> All]
```

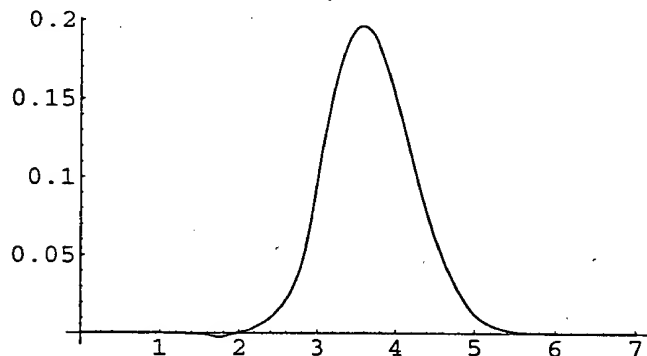
```
InterpolatingFunction::dmval : Input value {6.97422} lies outside the range of  
data in the interpolating function. Extrapolation will be used.
```

```
InterpolatingFunction::dmval : Input value {6.90039} lies outside the range of  
data in the interpolating function. Extrapolation will be used.
```

```
InterpolatingFunction::dmval : Input value {6.93624} lies outside the range of  
data in the interpolating function. Extrapolation will be used.
```

```
General::stop :
```

```
Further output of InterpolatingFunction::dmval will be suppressed during this calculation.
```



- Graphics -

? Modulator

Modulator[L][BitSeq,Opts] assumes the following default options
StartingQuadrant -> 0,InitialState -> Table[1,{i,1,20}],SamplingInterval -> T/32,
NumberOfCurves -> 4,ModulatingPulse-> LaurentC . The Pulse is assumed to have
the following structure Pulse[L][K][t]

? Interpolation

Interpolation[data] constructs an InterpolatingFunction object which represents an
approximate function that interpolates the data. The data can have the forms
{x1, f1}, {x2, f2}, ... } or {f1, f2, ... }, where in the second case, the
xi are taken to have values 1, 2,

```
Table[
```

```
Thread[var -> %247[[2]]]
```

```
Clear[Pulse]
```

```
Sinc[x_] /; x == 0 := 1
```

```
Sinc[x_] /; x != 0 := Sin[x] / (x)
```

```
PulseSampling := T/8
```

```
Pulse[8][0][y_] := x0[Round[y / PulseSampling] ];
```

```
Pulse[8][1][y_] := x1[Round[y / PulseSampling] ];
```

```
ModulationValue[FiltPulse[8]][{-1, -1, -1, -1, -1, -1, -1, -1}][0.5 T] // N
```

```
-0.754091 I
```

```
ConvertToBitSeq[Depth_] [Num_] :=
```

```
Module[{bit, n},
```

```
n = Num;
```

```
ConvertNext := (bit = Mod[n, 2]; n = Floor[n/2]; bit);
```

```
( Table[ConvertNext, {i, 1, Depth}] // Reverse) /. {0 -> -1}];
```

[illegible]

```
x1 = Map[ModulationValue[FiltPulse[8]][#][0.5 T]&,
  Select[Table[i, {i, 1, 2^8}] // Map[ConvertToBitSeq[8], #]&, (#[[5]] == 1)&] ]
  (-I)
```

```
{0.749266, 0.74922, 0.749173, 0.749219, 0.711482, 0.711529, 0.711482, 0.711435,
0.829796, 0.82975, 0.829703, 0.829749, 0.792012, 0.792059, 0.792012, 0.791965,
0.776885, 0.776839, 0.776791, 0.776838, 0.739101, 0.739147, 0.7391, 0.739054,
0.802178, 0.802131, 0.802084, 0.80213, 0.764394, 0.76444, 0.764393, 0.764347,
0.759521, 0.759475, 0.759428, 0.759474, 0.721738, 0.721784, 0.721737, 0.72169,
0.819541, 0.819495, 0.819447, 0.819494, 0.781757, 0.781804, 0.781756, 0.78171,
0.78714, 0.787094, 0.787047, 0.787093, 0.749356, 0.749403, 0.749355, 0.749309,
0.791922, 0.791876, 0.791829, 0.791875, 0.754138, 0.754185, 0.754138, 0.754091,
0.749266, 0.74922, 0.749173, 0.749219, 0.711482, 0.711529, 0.711482, 0.711435,
0.829796, 0.82975, 0.829703, 0.829749, 0.792012, 0.792059, 0.792012, 0.791965,
0.776885, 0.776839, 0.776791, 0.776838, 0.739101, 0.739147, 0.7391, 0.739054,
0.802178, 0.802131, 0.802084, 0.80213, 0.764394, 0.76444, 0.764393, 0.764347,
0.759521, 0.759475, 0.759428, 0.759474, 0.721738, 0.721784, 0.721737, 0.72169,
0.819541, 0.819495, 0.819447, 0.819494, 0.781757, 0.781804, 0.781756, 0.78171,
0.78714, 0.787094, 0.787047, 0.787093, 0.749356, 0.749403, 0.749355, 0.749309,
0.791922, 0.791876, 0.791829, 0.791875, 0.754138, 0.754185, 0.754138, 0.754091}
```

```
Limit[D[Sin[x]/x, x], x -> 0]
```

```
0
```

```
? Limit
```

```
Limit[expr, x->x0] finds the limiting value of expr when x approaches x0.
```

```
Map[(1/2 - σ/2 Erf[ $\frac{\#}{\sigma}$ ])&, x1] // Apply[Plus, #]& // #/128 &;
```

```
Ber[σ_] := Evaluate[%48]
```

```
Ber[0.01]
```

```
0.495
```

```
Erf[Infinity]
```

```
1
```

```
D[Erf[x], x]
```

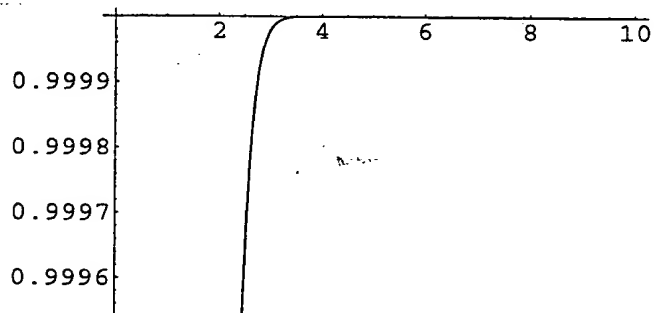
$$\frac{2 e^{-x^2}}{\sqrt{\pi}}$$

```
? Erf
```

```
Erf[z] gives the error function erf(z). Erf[z0, z1] gives the generalized error
function erf(z1) - erf(z0).
```



```
Plot[Erf[x], {x, 0, 10}]
```



- Graphics -

```
?FiltPulse
```

```
Global`FiltPulse
```

```
T := 3 / 812500
```

```
Null
```

```
?? T
```

This is the symbol period

```
?? T
```

This is the symbol period

```
T := 3 / 812500
```

```
Clear[T, x0, x1, x2, x3, x4, x5, x6, x7]
```

```

BeginPackage["LaurentFunctions`"]

T::usage = "This is the symbol period"

BT::usage = "This is the usual product"

h::usage = "This is the raw gaussian pulse"

 $\tilde{x}$ ::usage = "This denotes modulation index  $\pi$ "

 $\psi$ ::usage = " $\psi[L,t]$  is Laurents function"

hFiltered::usage = "This is the filtered gaussian pulse"

PhaseAngle::usage = "This function takes some time to calculate. The following
  code will draw a graph of the functionPhaseFunction[t_] = N[  $\phi_{L,t}$ ];
phasepoints = Table[{t,PhaseFunction[t T]}/N,{t,0,L ,1/40}];
ListPlot[phasepoints,PlotJoined -> True]"

S::usage = "S = Sin[ $\tilde{x}$ ]"

J::usage = "J =  $e^{j\tilde{x}}$ "

C::usage = "C = Cos[ $\tilde{x}$ ]"

M::usage = "M =  $2^{L-1}$ "

ModulationIndex::usage = "ModulationIndex = h"

LaurentS::usage = "LaurentS[L][n][t] = Sin[  $\psi[L,t + n T]$ ]/ S"

LaurentC::usage = "LaurentC[L][K][t] is Laurents  $C_{K,t}$ "

AlphaKI::usage = "AlphaKI[LL][K,i] is Laurents  $\alpha_{K,i}$ "

LaurentLK::usage = "LaurentLK[L][K] gives the support of  $C_{K,t}$ "

```

The start of Modulator Definitions

```

ANKInitialStateSetup::usage =
  "ANKInitialStateSetup[L][K][InitBitSeq,AccumulatedPhase] sets up the sequence
  of prior states of  $A_{K,n}$  that the modulator went thgough to get to the
  constellation point specified by AccumulatedPhase which is really  $A_{0,0}$ "

AKN::usage = "AKN[L][K][{State,AccumulatedPhase}] defines  $A_{K,n}$  in terms of  $A_{0,n}$ "

ModulatingPulse::usage =
  "The Pulse is assumed to have the following structure Pulse[L][K][t]"

NumberOfCurves::usage = "The number of pulses used by the modulator"

SamplingInterval::usage =
  "SamplingInterval is the interval between samples of the output of the modulator"

InitialState::usage = "InitialState is the set of bits that are assumed
  to be present before i.e { $a_1, a_2, \dots$ }"

StartingQuadrant::usage = "StartingQuadrant =  $A_{0,-1}$  and is a number"

Modulator::usage = "Modulator[L][BitSeq,Opts] assumes the following
  default options StartingQuadrant  $\rightarrow$  0, InitialState  $\rightarrow$  Table[1,{1,
  1,20}], SamplingInterval  $\rightarrow$  T/32, NumberOfCurves  $\rightarrow$  4, ModulatingPulse  $\rightarrow$ 
  LaurentC . The Pulse is assumed to have the following structure Pulse[L][K][t]"

```

Start of the Receiver Functions

```

FiltPulse::usage = "The default pulse and is called by FiltPulse[L][K][t]"

SyncSample::usage = "Given that the sampling interval is T/32, then sync
sample has rang -16 to 16 and this moves the point used to demodulate"

Receiver::usage = " Receiver[L][InputSeq,Opts] assumes the following
default options {StartingQuadrant->0,InitialState->{1,1,1,1,1,1,1,1,
1,1,1,1,1,1,1,1},SamplingInterval -> T/32,ModulatingPulse ->
FiltPulse,SyncSample -> 0,NumberOfCurves -> 2"

ReceiverProper::usage = "ReceiverProper[L][ StartingQuadrant, InitialState,
SamplingInterval,ModulatingPulse,SyncSample,NumberOfCurves,InputSeq]
is called by Receiver after all the options have been resolved"

Begin["`Private`"]

```

$$\sigma := \frac{\sqrt{\text{Log}[2]}}{2 \pi B T}$$

$$h[t_] := \frac{\text{Exp}\left[-\frac{t^2}{2 \sigma^2 T^2}\right]}{\sqrt{2 \pi} \sigma T}$$

Ideally we would and did define the effect of the convolution of $h[t]$ by the formula below. However, this version of *Mathematica* gives an error.

```

hFilt[t_] := Release[Module[{t},  $\int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{h[t-\tau]}{T} d\tau$ ]]

hFiltered[PulseWidth_][t_] := Module[{x1, x2, x3}, x1 =
(N[#, 40] &)[Table[{t1,  $\int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{h[t1-\tau]}{T} d\tau$ }, {t1, - $\frac{\text{PulseWidth}}{2}$ ,  $\frac{\text{PulseWidth}}{2}$ ,  $\frac{T}{20}$ }}]],
x2 = Interpolation[x1]; x2[t]]

x := N[ModulationIndex  $\pi$ ]

C := Cos[x];
S := Sin[x];
J := (ej x // Chop);
M := 2L-1;

PhaseAngle[L_][t_] /; t < 0 := 0

PhaseAngle[L_][t_] /; t > L T := x

PhaseAngle[L_][t_] :=
PhaseAngle[L][t_] = Module[{x1, x2, x3, x4, x5, x6}, x1 = hFiltered[3 L T][t1 -  $\frac{L T}{2}$ ],
x2 = Table[{t2,  $x \int_{-L T}^{t2} \text{Evaluate}[x1] dt1$ }, {t2, 0, L T,  $\frac{T}{100}$ }}]; Interpolation[x2][t]]

 $\psi[L_, t_] /; 0 < t < L T := \text{PhaseAngle}[L][t]$ 

 $\psi[L_, t_] /; 2 L T > t \geq L T := x - \text{PhaseAngle}[L][t - L T]$ 

```

We need to put this to avoid the function being extrapolated

```

 $\psi[L_, t_] /; ! (0 < t < L T) \&\& ! (2 L T > t \geq L T) := 0$ 

LaurentS[L_][n_][t_] := Sin[ $\psi[L, t + n T]$ ] / S

```

```

AlphaKI[LL_][K_, i_] /; (0 < i < LL) && (0 <= K < 2LL-1) :=
Module[{x1, x2, x3, KNum}, x1 := (x2 = Mod[KNum, 2]; KNum =  $\frac{KNum - x2}{2}$ ; x2);
KNum = K; x3 = Table[x1, {ii, 0, LL - 1}]; x3[[i]]

LaurentLK[L_][K_] :=
Module[{x1}, x1 = Table[L (2 - AlphaKI[L][K, ii]) - ii, {ii, 1, L - 1}]; Min[x1]]

LaurentC[L_][K_][t_] /; 0 <= K < 2L :=
LaurentS[L][0][t]  $\prod_{ii=1}^{L-1}$  LaurentS[L][ii + L AlphaKI[L][K, ii]][t]

```

The start of the modulator function

```

ANKInitialStateSetup[L_][K_][InitBitSeq_, AccumulatedPhase_] :=
Module[{x1, x2, x3, x4, x5, acuphase, initbitseq},
initbitseq = InitBitSeq;
acuphase = AccumulatedPhase;
UpdateSeq :=
Module[{}], x1 = acuphase - Sum[initbitseq[[i]] AlphaKI[L][K, i], {i, 1, L - 1}];
acuphase = acuphase - First[initbitseq]; initbitseq = Rest[initbitseq]; x1];
Table[UpdateSeq, {i, 1, LaurentLK[L][K]}]]

AKN[L_][K_] [{State_, AccumulatedPhase_}] :=
AccumulatedPhase - Sum[State[[i + 1]] AlphaKI[L][K, i], {i, 1, L - 1}]

Options[Modulator] := {StartingQuadrant -> 0, InitialState -> Table[1, {i, 1, 20}],
SamplingInterval -> T/32, NumberOfCurves -> 4, ModulatingPulse -> LaurentC}

Modulator[L_][BitSeq_, Opts_] :=
Module[
{x1, x2, x3, x4, x5, x6, state, AccumulatedPhase, seq, AKNState, Curves, Pulse},
x1 = SamplingInterval /. {Opts} /. Options[Modulator];
state = InitialState /. {Opts} /. Options[Modulator];
x3 = StartingQuadrant /. {Opts} /. Options[Modulator];
x4 = SamplingInterval /. {Opts} /. Options[Modulator];
Pulse = ModulatingPulse /. {Opts} /. Options[Modulator];
Curves = (NumberOfCurves /. {Opts} /. Options[Modulator]) - 1;
AccumulatedPhase = x3;
seq = BitSeq;
Table[
AKNState[K] = ANKInitialStateSetup[L][K][state, AccumulatedPhase], {K, 0, Curves}];
x5 := Module[{}], state = Join[{First[seq]}, Drop[state, -1]];
AccumulatedPhase = AccumulatedPhase + First[seq];
seq = Rest[seq];
Table[AKNState[K] = Join[{AKN[L][K] [{state, AccumulatedPhase}]},
Drop[AKNState[K], -1]], {K, 0, Curves}];
x6[t_] = Sum[Sum[(J)AKNState[K] [[i + 1]] Pulse[L][K][t + i T],
{i, 0, LaurentLK[L][K] - 1}], {K, 0, Curves}];
Table[x6[t], {t, 0, T - x4, x4}];
Table[x5, {kk, 1, Length[BitSeq]}] // Flatten

Options[Receiver] := {StartingQuadrant -> 0,
InitialState -> {1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}, SamplingInter
ModulatingPulse -> FiltPulse, SyncSample -> 0, NumberOfCurves -> 2};

Receiver[L_][InputSeq_, Opts_] :=
Module[{x1, x2, x3, x4, x5, x6},
x1 = StartingQuadrant /. {Opts} /. Options[Receiver];
x2 = InitialState /. {Opts} /. Options[Receiver];
x3 = SamplingInterval /. {Opts} /. Options[Receiver];
x4 = ModulatingPulse /. {Opts} /. Options[Receiver];
x5 = SyncSample /. {Opts} /. Options[Receiver];
x6 = NumberOfCurves /. {Opts} /. Options[Receiver];
ReceiverProper[L][x1, x2, x3, x4, x5, x6, InputSeq]]

```

```

ReceiverProper[L_][ StartingQuadrant_, InitialState_, SamplingInterval_,
  ModulatingPulse_, SyncSample_, NumberOfCurves_, InputSeq_] :=
  Module[{x1, sgn, ReceivedSeq, ExpectedValue, seq, ReceiveNext, D, J},
    x1 = T / SamplingInterval;
    ReceivedSeq = Partition[InputSeq, x1] // Transpose // #[[SyncSample + 1]] &;
    ExpectedValue =
      ModulatingPulse[L][0][ (LaurentLK[L][0] / 2) T + SyncSample SamplingInterval];
    J = StartingQuadrant;
    sgn = 0;
    D = {};
    seq = ReceivedSeq;
    ReceiveNext :=
      Module[{x1, x2},
        x1 = ( (-1)sgn JJ First[seq] ) // Im;
        If[Abs[x1 - ExpectedValue] <= Abs[x1 + ExpectedValue],
          D = Join[ D, {1}]; J = J + 1, D = Join[ D, {-1}]; J = J - 1 ];
          seq = Rest[seq]; sgn = Mod[sgn + 1, 2];
        Table[ReceiveNext, {i, 1, Length[ReceivedSeq]}];
        D]
  ]

End[]

EndPackage[]

```